Outline of a book on Statistical and Quantum Optics

1 *Statistical and Quantum Optics* © R. M. Sillitto 1977

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Chapter 1

1.1.1 The concepts of quantum physics ...

Quantum optics began when optics began. Indeed quantum physics began with quantum optics. Planck's interpretation of Lummer and Pringsheim's measurements of the spectral intensity distribution in black body radiation (1899; see Appendix 1.1) introduced Planck's constant into physics, and the idea of quantization followed from this. Then Einstein used the idea of the quantum of radiant energy – later named the *photon* – to interpret the observations of the photoelectric effect by Hertz (1887; see Appendix 1.2). Thereafter the development of quantum theory leaned heavily on evidence provided by optics, but this development seemed to contribute surprisingly little fresh insight into optical phenomena *per se*.

Perhaps this was as well. Progress in theoretical atomic physics and in quantum theory was very dependent on evidence provided by optical spectroscopy. As long as the development of quantum theory did not seem to call for any fundamental reinterpretation of optical processes, the optical evidence could be relied upon, and it stimulated and guided the quantum theorists at many critical stages of their work. Even though the need for a quantum theory had become evident in the context of interactions between matter and radiation, the quantum theorists' task in the first quarter of this century turned out to be the construction of a quantum theory of material systems, and the theoretical basis of physical optics was not seriously called in question at that stage. (In passing, it is interesting to note that physical optics survived almost unscathed that other revolution in theoretical physics, the advent of the special theory of relativity.)

This is not to say that quantum theory raised no problems in the field of optics. If a naive 'photon model' of light were valid, the phenomena of interference and diffraction would have to be rather drastically reinterpreted. But the capacity of the wave theory to predict the results of experiments in these areas was well known, and the crucial experiments on diffraction and interference at extremely low intensities which were performed by Taylor (1909; see Appendix **??**), by Dempster and Batho (1927; see Appendix **??**), and by others, gave results reassuringly similar to those already familiar at higher intensities: diffraction and interference effects are not intensity-dependent. Whether the energy density in the light field propagating through the apparatus is large or small, the diffraction or interference pattern predicted by classical optics is evidently the probability-density-in-position for the arrival of a photon on the screen, photographic plate, or photocathode where the light is observed. From this, Dirac argued (see *Principles of Quantum Mechanics*, chapter 1) that the solution of Maxwell's equations in classical electromagnetic theory must be exactly equivalent to the solution of the quantum/wave equation for a single photon. This ensures that the classical theory describes correctly the propagation of light fields in homogeneous media in which the electric and magnetic polarisations are proportional to the driving fields: though of course, at the *microscopic* level the only homogeneous medium is free space! The problem of the interaction of matter and radiation is a very complex one, and requires a complete revision of the classical electron theory of Lorentz and others. To start with, this is principally a matter of developing a quantum theory of material systems. The phenomena which can be discussed in relation to quite simple material systems reveal a great deal of what is interesting and fundamental about quantum theory; and the interaction between radiation and matter is easier to formulate and understand once the characteristically quantum properties of atoms and molecules are familiar.

1.1.2 . . . and of optical coherence . . .

The development of the formal theory of quantum mechanics in the late 1920s soon led to a quantum theory of radiation, with which many aspects of the radiation-matter interaction could be discussed. However, for optical purposes we have to be able to discuss the interaction of radiation fields with light sources and photodetectors, and one has to inquire into the statistical characteristics of the radiation fields generated by light sources containing many excited atoms or molecules. This inquiry originated with the work of Fizeau, and Verdet, in 1865. It was Fizeau who first suggested the association, between source size and the observability of interference fringes, which underlies Michelson's technique for measuring the sizes of stars (Michelson, 1890; see Appendix **??**); Verdet studied the size of the region of coherence for light derived from an extended source. In 1891, the year after his first description of the stellar interferometer, Michelson established the connection between the spectral energy distribution of a light source and the visibility of two-beam interference fringes when the light paths are unequal (see Appendix **??**). Michelson thus provided, with his two famous interferometers, the tools for measuring the spatial and temporal coherence properties of a light beam.

The coherence properties of a light beam result from the stochastic behaviour of the source. The very fundamental nature of this behaviour was emphasised by von Laue, who introduced a measure of the "degree of coherence" in the course of a discussion of the thermodynamics of light fields, published in 1907; and in 1915 there was some discussion in print between Einstein and von Laue, as to the appropriate probability density function for the amplitude of the field in black body radiation. This latter aspect of the statistical distribution of light fields was developed further by von Cittert, in two papers published in 1934 and 1939, in which he determined joint probability density functions for (i) the simultaneous light amplitudes at any two points in a plane illuminated by an extended source ("spatial coherence") and (ii) the light amplitudes at a single point at two different times when the light source has a finite spectral bandwidth ("temporal coherence"). Zernike (1938) independently related the spatial coherence of a light beam to the size of the source, showed that the degree of coherence of a radiation field can be thought of as a propagating characteristic of the field, and in particular showed how the degree of coherence propagates through optical instruments. The application of coherence theory to imaging was subsequently powerfully developed by Hopkins and others.

Through these investigations the notion of partial coherence was clearly and quantitatively established, but the conclusions were applicable only to stationary, ergodic monochromatic fields and smallish path differences. The generalisation to polychromatic fields and longer path differences was made independently by Wolf (1954) and by Blanc-Lapierre and Dumontet(1955). In particular, both developed the theory of optical coherence into a theory of physical optics in terms of observables. This was a profoundly important development. The elementary theory of physical optics deals with field amplitudes and time variations which are not observable. The coherence theory, on the other hand, deals with quantities – correlation functions which are measured by the visibility of interference fringes, and of which the ensemble-averaged intensity is a special case – which *are* observable, and whose propagation accounts for all the observable effects in physical optics. When physical optics was represented in this way it was ripe for quantisation, and two experiments reported in 1955 and 1956 made it urgently necessary to develop a complete quantum theory of the optical radiation field interacting with its sources and detectors. The first of these experiments, by Forrester, Gudmundsen and Johnson (see Appendix **??**) demonstrated the generation of beats – or, more strictly, a difference tone – between spectrally resolved Zeeman components of the 546.1 nm mercury green line; the second, by Hanbury Brown and Twiss (see Appendix **??**), demonstrated correlations in time between the photoelectrons from detectors at different points in a light field. The performance of both these experiments depended on the availability of photoelectric detectors in which the instants when photoelectric emissions occurred could be timed with considerable precision: the description of the photoelectric emission process requires a quantum theory. But the phenomena which these experiments demonstrated are described by the kind of correlation functions of the light field which occur in the classical theory of coherence. So a complete description of these experiments required a merging of the coherence theory of light with the quantum theory of the photoelectric effect. Attempts to think about the Forrester Gudmundsen Johnson effect and the Hanbury Brown Twiss effect in terms of photon models of light – what Jaynes has recently called the "buckshot model" – were confusing, and gave rise to futile controversies as to whether or not the effects "really exist". And then the invention of the laser – in which highly monochromatic and coherent light was generated by macroscopic assemblies of ions or atoms (Maiman (1960); Javan, Bennett and Herriott (1961); see Appendix **??**) showed the need for a deeper consideration of the stochastic properties of light.

1.1.3 . . . are brought together in quantum optics

The theoretical basis of quantum optics was established by Glauber (1963) – who brought together quantum theory and optical coherence in a "quantum theory of optical coherence" – and by Sudarshan (1963), who emphasised how close was the equivalence between the quantum and classical theories of optics for classically visualisable fields. The merging of ideas from statistics, electromagnetic theory and quantum theory leads to a description of quantised electromagnetic fields which generalises the classical theory of optical coherence, and makes it possible to discuss in detail not only interference phenomena but also the intensity correlation effects discovered in the 1950s, and the photon-counting experiments which have followed from them. The explicit incorporation of statistical ideas allows us to interpret the different quality of laser light and of "natural" or "thermal" light in terms of the different probability-densities-in-amplitude which embody the stochastic character of the light fields, and from this we can relate the coherence properties of these different light fields to the counting statistics which they give rise to in photon-counting experiments.

The last 15 years[1], in which, to quote Glauber, “the insight of quantum electrodynamics has been brought to bear on the problems of optics”, has been a period of brilliant advance in both theoretical and experimental optics. Our understanding of the nature of light has been deepened and enriched, and new optical techniques have emerged to contribute to fields as diverse as astronomy, civil engineering, and molecular biology. Once again optics, one of the oldest branches of natural philosophy, has proved to be one of the freshest and most exciting branches of physics.

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[1] since 1962

Chapter 2: The random nature of light

Morphology is always the beginning

C. F. Von Weizsacker (1951)

The theory of optical coherence is concerned with the fluctuations of the field strength, intensity, and so on, in beams of light, and describes the correlations which can be observed between the fields at different points and different times. The fluctuations arise in the first instance from the random nature of the radiation process and the light source, and the ideas of the theory of coherence can therefore be used to relate the characteristics of a light source to the statistical variations in the light it emits, whether these be variations in space, in time, or both. The fluctuations in the field are modified as the light propagates: propagation through an inhomogeneous medium, such as a turbulent atmosphere, produces fluctuations – in the intensity and phase of the light - – which are characteristic of the medium; the passage of light through a lens modifies the coherence of the light in a way which is characteristic of the lens, and is related to its imaging properties. In this and the two following chapters we shall be concerned with certain aspects of the classical theory of coherence, whose quantum mechanical counterpart will be introduced in Chapter 8.

Chapter 3: The spectral analysis of random signals

> *The multiplicity of vibrational forms which can be thus produced by the composition of simple pendular vibrations is not merely extraordinarily great: it is so great that it cannot be greater.*
>
> *Hermann L. F. Helmholtz (1877) (1951)*

The spectral analysis of periodic and nonperiodic signals whose form is known is familiar in many branches of physics and the synthesis of a specified waveform by the superposition of harmonic components is often demonstrated, for instance in acoustic and electronic contexts. An optical spectroscope, however, performs a rather subtler kind of frequency analysis. The optical radiation field is a random process, so the optical spectroscope must be an instrument for frequency-analysing random signals; and the stability of the spectra it produces shows that the power spectrum of the radiation field is one of the characteristic and persistent features of the random signal.

In this chapter the familiar processes of the Fourier analysis of periodic and nonperiodic signals will be reviewed briefly, and then the spectral analysis of random signals and of optical signals in particular, will be discussed at greater length

Chapter 4: Optical coherence: interference, coherence, and intensity correlation

> *The whole science of optics . . . has just consisted of the constructive use of noise*
>
> *R. J. Glauber (1969)*

In this chapter the general discussion of random processes in chapters 2 and 3 will be particularised and applied to the discussion of a number of the characteristic effects of classical physical optics, such as two beam interference, diffraction, and imaging. These are all large studies in their own right, and the purpose of this chapter is simply to show how they are all linked together as examples of the behaviour of light waves regarded as random processes.

Chapter 5: Quantum theory of an oscillator

> *A warld this of probability,*
> *states and transitions. The causality*
> *we kent at hame that let us understaund*
> *acceleration, orbit, trajectory,*
> *we maun leave ahint as contraband*
> *to pass the frontier of this fremmit land.*
> *Quantum of action is the new passport*
> *that gies us pouers of correspondency.*
>
> *Robert Garioch (1963)*

The one-dimensional harmonic oscillator is discussed in most introductory courses on quantum mechanics, and in most quantum mechanics texts. Nevertheless, its properties are so fundamental to quantum optics, and the notation in which they are described is such a basic part of the language of the subject, that it is worth starting the quantum part of this book with an account of the quantum theory of the oscillator. In any case, there are aspects of this topic which have not usually figured in introductory accounts but which are necessary for the discussion of optical problems – notably the coherent states – which can conveniently be introduced in the course of the general survey of the quantum theory of the oscillator. The Dirac notation will be used: the reader unfamiliar with it will find an excellent introduction to its use in P. T. Matthews *Introduction to quantum mechanics* (McGraw-Hill) 1963. It is important to acquire facility with this very elegant and powerful notation, since it is used throughout the current literature of quantum optics.

Chapter 6: The quantization of radiation

> *. . . to hold two opposed ideas in the mind at the same time, and still retain the ability to function.*
>
> *F. Scott Fitzgerald (1936)*

So far in this book the electromagnetic field of light beams has been viewed as a classical wave system. But historically the interaction between light and matter, often epitomised by the problem of optical dispersion, was the rock on which theories of radiation foundered, up to and including the Bohr-Sömerfeld theory. In order to tackle the problem of the photoelectric effect, and the detection of light, quantum mechanics has to be merged with the electromagnetic theory, and the way in which this is done will be a recurring theme in the subsequent chapters.

Chapter 7: Assemblies: the density operator, and light beams

> *If we follow through the logic of a fundamentally probabilistic view of the world, it becomes less strange, and almost what we should expect*
>
> *J. R. Lucas (1970)*

Chapter 8: The detection of light, and first-order coherence

Stars, I have seen them fall,
But when they drop and die
No star is lost at all
From all the star-sown sky.

A. E. Housman (1936)

In classical physical optics the interaction of light with the detector was scarcely considered; the implication of this is that the detector is ‘slow’, that is, that its response time is long compared with the coherence time of the light. As long as light sources were ‘thermal’, and detectors were the eye, the photographic plate, or the photocell used with a narrow band amplifier, slowness of response was indeed assured. The advent of the photo-multiplier tube, the image intensifier, and nanosecond circuitry has now made it possible to work with response times not much longer than the coherence times of carefully designed thermal light sources (see, e.g., appendix A.xx), and the invention of the laser provided a light source whose coherence time could be longer than the response time of relatively simple circuitry. The detection of a single quantum – or, more precisely, the emission of a single photo-electron from a cathode – can now be recorded. This is a quantum mechanical event, so it is necessary to discuss the interaction between radiation and matter in quantum mechanical terms.

Chapter 9: Higher order coherence, and photon statistics

> *To think that two and two are four*
> *And neither five nor three*
> *The heart of man has long been sore*
> *And long 'tis like to be*
>
> *A. E. Housman (1922)*

The properties of the most general type of radiation field can be described in terms of a hierarchy of correlation functions; the experiments which measure these correlation functions are either multi-detector coincidence experiments, or single-counter experiments which find the relative frequencies of the detection of 1, 2, 3… quanta in counting intervals of fixed duration. Starting with the Hanbury Brown – Twiss experiment, the study of higher order correlations and of photocounting distributions has developed rapidly and the techniques of 'photon statistics' have found applications in many branches of physics. Some of the basic ideas will be discussed in this chapter, and chapter 10 will describe some recent applications of these ideas.

Chapter 10: Applications of photon counting statistics

A pattern is a message, and may be transmitted as a message

Norbert Wiener (1954)

The first experiments in the then-unrecognised field of photon-statistics were those of Forrester, Gudmundsen and Johnson (Appendix **??**) and Hanbury Brown and Twiss (Appendix **??**). The first of these experiments measured the temporal fluctuations of the light-field, the second measured the spatial fluctuations. In both cases very complex, highly specialised circuitry was used for recording and analysing currents produced in photoelectric detectors. Within a matter of three or four years, photomultiplier tubes responding to single quanta were being used in conjunction with fast coincidence circuits to measure the second-order correlation function more directly. For thermal light sources, in which all the correlation functions can be expressed in terms of the first-order correlation, two-fold coincidence experiments convey as much information as multiple-coincidence experiments would do. This is fortunate since the two-fold coincidence counting-rates in circuits with nanosecond resolving times are so low that higher-order coincidence experiments could not be contemplated.

The advent of the laser, with its high photon-number beams and long coherence times – milliseconds or seconds in the case of gas lasers – created the need for methods of investigating higher-order correlations. The power of the range of application of this technique is so great that it can only be hinted at here. With biological materials, studies of translational and rotational Brownian motion have been carried out in which the spectral line-width resulting from the scattering of laser light from the randomly moving macromolecules is no more than a few Hertz – corresponding to speeds of about 10^{-5} cm s^{-1} in the case of the translational Brownian motion, and optical resolving powers of about 10^{15}. At the other extreme, a two-fold delayed coincidence experiment has been used to measure Doppler-broadened spectral line widths of about 10^{8} Hz from mercury discharges. Studies of the photon statistics of laser light have been used to test laser theories, and the transition from thermal light to fully coherent light as the power input to the laser is increased has been followed through in detail; the factorisation of the correlation functions up to sixth order has been demonstrated for laser light. Laser light scattered from turbulent gas flows, from matter near its critical point, from biological materials in liquid suspensions, and from chemical reactions, has been shown to acquire the chaotic characteristics of thermal light; the measurement of the coherence volumes and coherence times reveals features of these phenomena – their spatial scale and relaxation time constants, in particular – which are frequently not as accessible by other means.

This chapter will indicate briefly how some of these studies relate to the ideas and methods developed in the earlier chapters; a brief bibliography of recent review papers dealing with photon statistics ends the chapter.